Lab 18: Section 13.2 Testing the Difference Between Means (Dependent Samples)

In Section 13.1, you performed two-sample hypothesis tests with *independent samples* using the test statistic $\bar{x}_1 - \bar{x}_2$ (the difference between the means of the two samples). To perform a two-sample hypothesis test with *dependent samples*, you will use a different technique. You will first find the difference d for each data pair.

$$d = \text{(data entry in first sample)} - \text{(corresponding data entry in second sample)}$$

The ***test statistic*** is the mean $\bar{d}$ of these differences

$$\bar{d} = \frac{\Sigma d}{n} \qquad \bar{d} = \frac{\Sigma d}{n}$$

That is to say that d bar is the average of the differences between paired data entries in the dependent samples

The ~~***standardized***~~ ***test statistic*** is

Where n equals the number of pairs of data

test statistic → $\boxed{t = \dfrac{\bar{d} - \mu_d}{s_d/\sqrt{n}}}$

The degrees of freedom are

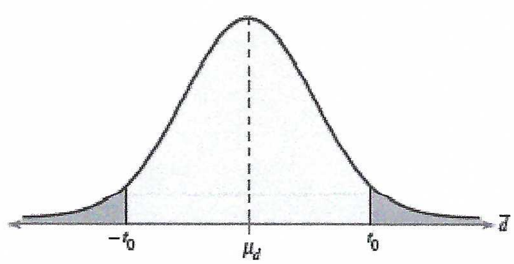
$$d.f. = n - 1.$$

These conditions are necessary to conduct the test.

1. The samples are randomly selected.
2. The samples are dependent (paired).
3. The populations are normally distributed or the number n of pairs of data is at least 30.

When these conditions are met, the sampling distribution for $\bar{d}$, the mean of the differences of the paired data entries in the dependent samples, is approximated by a t-distribution with $n - 1$ degrees of freedom, where n is the number of data pairs.

As a result, we can use the calculator's t-test function to get the p-value and the value of the standardized test statistic.



The symbols listed in the table (below) are used for the t-test for μ_d. Although formulas are given for the mean and standard deviation of differences, you should use technology to calculate these statistics.

The t-test Calculator command

Symbol	Description
n	The number of pairs of data
d	The difference between entries in a data pair
μ_d	The hypothesized mean of the differences of paired data in the population
$\bar{d}$	The mean of the differences between the paired data entries in the dependent samples $\bar{d} = \frac{\Sigma d}{n}$
s_d	The standard deviation of the differences between the paired data entries in the dependent samples $s_d = \sqrt{\frac{\Sigma(d - \bar{d})^2}{n - 1}}$

Classroom Exercises

1. A medical researcher wants to determine whether a drug changes the body's temperature. Seven test subjects are randomly selected, and the body temperature (in degrees Fahrenheit) of each is measured. The subjects are then given the drug and, after 20 minutes, the body temperature of each is measured again. The results are listed below. At $\boxed{\alpha = 0.05}$, is there enough evidence to conclude that the drug changes the body's temperature? Assume the body temperatures are normally distributed.

	Subject	1	2	3	4	5	6	7
L1	Initial temperature	101.8	98.5	98.1	99.4	98.9	100.2	97.9
L2	Second temperature	99.2	98.4	98.2	99.0	98.6	99.7	97.8
L3 = L1 − L2		2.6	0.1	−0.1	0.4	0.3	0.5	0.1

$\bar{d}$ is the mean of this list

(a) (2 points) Write the null and alternative hypotheses.

$H_0: \mu_d = 0$ (no change)

(two-tailed) → $H_A: \mu_d \neq 0$ (changes the Body Temp)

(b) (2 points) What conditions should you check first before you conduct the hypothesis test?

1. The 2 samples are paired.
2. The n sample differences can be viewed as a random (or representative) sample from a population of differences.
3. The number of sample differences is large $n \geq 30$ OR the pop. of differences is approx. normal.

(c) (1 point) What formula should be used for the test statistic?

$$\boxed{t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}}$$

(d) (1 point) What number is the test statistic equal to?

t-test on calculator now $\boxed{t = 1.596}$

(e) (1 point) What p-value do you obtain? Round to the ten-thousandths.

$$\text{p-val} = 2 \cdot P(\bar{d} > 0.557 \text{ assuming } \mu_d = 0) = 2 \cdot P(t > 1.596) = \boxed{0.1616}$$

(f) (1 point) Do you reject the null hypothesis or fail to reject the null hypothesis? Explain.

$\alpha = 0.05$ | Since the p-val > α, fail to reject H_0

(g) (2 points) Please write a conclusion sentence in the context of the problem.

There is not convincing sample evidence to suggest that the drug changes the body temperature.

2. ***Pneumonia*** A scientist claims that pneumonia causes weight loss in mice. The table below shows the weights (in grams) of six randomly selected mice before infection and two days after infection. At $\alpha = 0.01$, is there enough evidence to support the scientist's claim? Assume that weights are normally distributed.

Mouse	1	2	3	4	5	6
Weight (before)	19.8	20.6	20.3	22.1	23.4	23.6
Weight (after)	18.4	19.6	19.6	20.7	22.2	23.0

(a) (2 points) Write the null and alternative hypotheses.

$H_0: \mu_d = 0$ (no weight change)

right-tailed $H_A: \mu_d > 0$ (weight loss occurs)

(b) (2 points) What conditions should you check first before you conduct the hypoth-

Same as those listed in problem 1, b.

esis test?

(c) (1 point) What formula should be used for the test statistic?

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$$

(d) (1 point) What number is the test statistic equal to?

$t \approx 7.456$ standard dev. lengths

(e) (1 point) What p-value do you obtain? Round to the ten-thousandths.

p-val $= P(\bar{d} > 1.05$, assuming $\mu_d = 0) = P(t > 7.456) = 3.4 \times 10^{-4}$ $= \boxed{0.0003}$

(f) (1 point) Do you reject the null hypothesis or fail to reject the null hypothesis? Explain.

$\alpha = 0.01$ / Since the p-val $\leq \alpha$, reject H_0

(g) (2 points) Please write a conclusion sentence in the context of the problem.

There is convincing sample evidence to support the scientists claim.